1. Velocity and Acceleration

Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Tangential velocity $v_t = r\omega$

Tangential acceleration $a_t = r\alpha$

Centripetal acceleration $a_n = v^2/r = \omega^2 r$

2. Constant acceleration

$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + vt + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

3. Moment of Inertia

Systems of particles $I = \sum m_i r_i^2$

Continuous object $I = \int r^2 dm$

Parallel-axis theorem $I = I_{cm} + Md^2$

4. Torque

$\tau = F_t r = Fr\sin\varphi = Fl$

5. Newton's Second law

Particle	Plane-parallel motion
$\sum \vec{F} = m\vec{a}$	$\begin{cases} F_{ext-x} = ma_{cm-x} \\ F_{ext-y} = ma_{cm-y} \\ \tau_{ext-cm} = I_{cm}\alpha \end{cases}$

6. Non-slip Condition

Rolling without slipping

$v_{cm} = r\omega$

$a_{cm} = r\alpha$

Point P with position $\vec{r}$ on the rigid body

$\vec{v}_p = \vec{v}_{cm} + \vec{\omega} \times \vec{r}$

If the velocity of the contact point A is zero

$v_{cm} = \omega r$

7. Work and energy

Kinetic energy $E_k = \frac{1}{2}mv^2$	Kinetic energy $E_k = \frac{1}{2}I\omega^2$
Power $P = FV$	Power $P = \tau\omega$
	Kinetic energy for rotating object $E_k = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

Work-energy theorem for rigid body:

$$\Delta E_k = E_k(t) - E_k(0) = \int_0^t \vec{F}_{ext} \cdot d\vec{r} + \int_{\theta_0}^{\theta} \tau_{ext-cm}\, d\theta$$

F_{ext} on its center of mass

τ_{ext} on its center of mass

8. Angular Momentum

Momentum	Angular Momentum
$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times \vec{p}$ $(kg \cdot m^2/s)$

Theorem of angular momentum:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Net torque act on the object equals to the change rate of angular momentum of the object about the same point.

Integral representation:

$$\Delta\vec{L} = \int_0^t \vec{\tau}_{net} dt$$

Angular impulse:

$$\vec{J} = \int_0^t \vec{\tau}_{net} dt \Rightarrow d\vec{L} = d\vec{J}$$

The change of angular momentum of the object equals to the angular impulse exert on the object about the same point.

Conservation of Angular Momentum

$$\vec{\tau}_{net} = \frac{d\vec{L}_{sys}}{dt} = 0 \Rightarrow \vec{L}_{sys} = constant$$

1. Velocity and Acceleration	
Velocity $v = dx/dt$ Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	Angular velocity $\omega = d\theta/dt$ Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential velocity $v_t = r\omega$ Tangential acceleration $a_t = r\alpha$ Centripetal acceleration $a_n = v^2/r = \omega^2 r$	

2. Constant acceleration	
$v = v_0 + at$ $x = x_0 + vt + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a(x - x_0)$	$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

3. Moment of Inertia
Systems of particles $I = \sum m_i r_i^2$ Continuous object $I = \int r^2 dm$ Parallel-axis theorem $I = I_{cm} + Md^2$

5. Torque
$\tau = F_t r = Fr\sin\varphi = Fl$

5. Newton's Second law	
Particle $\sum \vec{F} = m\vec{a}$	Plane-parallel motion $\begin{cases} F_{ext-x} = ma_{cm-x} \\ F_{ext-y} = ma_{cm-y} \\ \tau_{ext-cm} = I_{cm}\alpha \end{cases}$

6. Non-slip Condition
Rolling without slipping $v_{cm} = r\omega$ $a_{cm} = r\alpha$ Point P with position $\vec{r}$ on the rigid body $\vec{v}_p = \vec{v}_{cm} + \vec{\omega} \times \vec{r}$ If the velocity of the contact point A is zero $v_{cm} = \omega r$

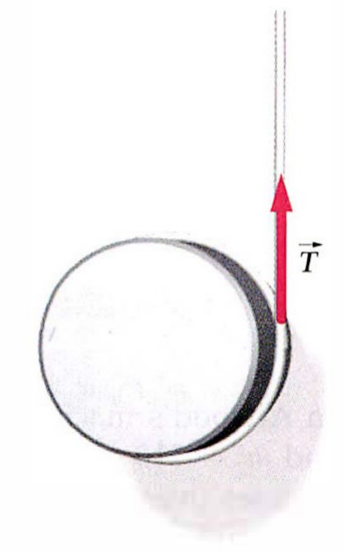
7. Work and energy	
Kinetic energy $E_k = \frac{1}{2}mv^2$ Power $P = FV$	Kinetic energy $E_k = \frac{1}{2}I\omega^2$ Power $P = \tau\omega$ Kinetic energy for rotating object $E_k = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$
Work-energy theorem for rigid body: $\Delta E_k = E_k(t) - E_k(0) = \int_0^t \vec{F}_{ext} \cdot d\vec{r} + \int_{\theta_0}^{\theta} \tau_{ext-cm}\, d\theta$ F_{ext} on its center of mass　　τ_{ext} on its center of mass	

8. Angular Momentum	
Momentum $\vec{p} = m\vec{v}$	Angular Momentum $\vec{L} = \vec{r} \times \vec{p}$ $(kg \cdot m^2/s)$
Theorem of angular momentum: $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ Net torque act on the object equals to the change rate of angular momentum of the object about the same point. Integral representation: $\Delta\vec{L} = \int_0^t \vec{\tau}_{net} dt$ Angular impulse: The change of angular momentum of the object equals to the angular impulse exert on the object about the same point. $\vec{J} = \int_0^t \vec{\tau}_{net} dt \Rightarrow \Delta\vec{L} = \vec{J}$	
Conservation of Angular Momentum $\vec{\tau}_{net} = \frac{d\vec{L}_{sys}}{dt} = 0 \Rightarrow \vec{L}_{sys} = constant$	

努力事项

- 书写工整些：阅卷者才能耐心看你的答案
- 需要有过程：光有结果很多分得不到。

Exercise 1:
The string wrapped around the cylinder of mass m in the right Figure is held by a hand that is accelerated upward so that the center of mass of the cylinder does not move. Find (a) the tension in the string.

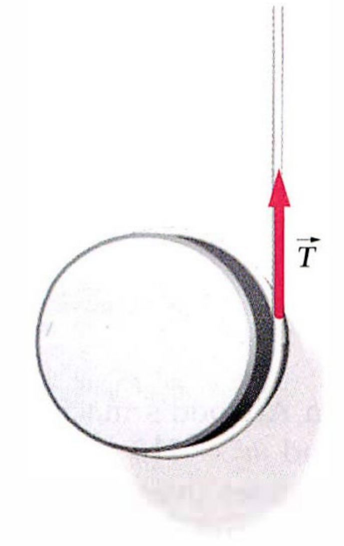


(b) the angular acceleration of the cylinder.

(c) the acceleration of the hand.

Exercise 2: A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed, and the cylinder falls vertically as shown in the right figure.

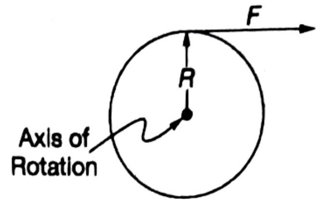
(a) Find the acceleration of the cylinder



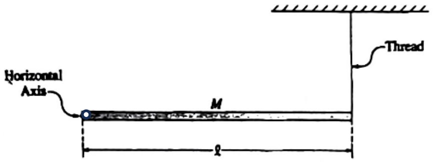
(b) Find the tension in the string.

Exercise 3: A uniform sphere rolls without slipping down an incline. What must be the angle of the incline if the linear acceleration of the center of mass of the sphere is 0.2g?

下面是错题整理 5. A solid disk of mass M, radius R and rotational inertia I is free to rotate about a fixed frictionless axis that is perpendicular to the disk through its center, as shown in the figure. A force of constant direction and constant magnitude F is exerted on the disk. If the disk accelerates from rest at time t=0, through what angle does the disk rotate from t=0 to t=T?



7 A long, uniform rod of mass M and length l is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown in the figure. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of M, l, and g.



a. Determine the magnitude and the direction of the force exerted on the rod by the axis.

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following.

b. The angular acceleration of the rod about the axis.

c. The translational acceleration of the center of mass of the rod.

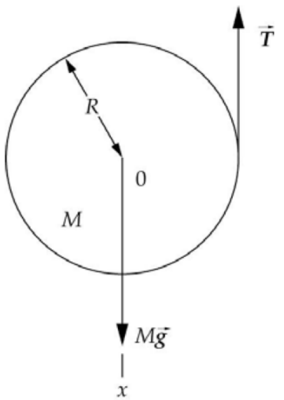
d. The force exerted on the end of the rod by the axis.

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod as swung.

答案 Exercise 1

Picture the Problem Choose the coordinate system shown in the diagram. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T and a. In (b) we can use the torque equation from (a) and our value for T to find α. In (c) we use the condition that the acceleration of a point on the rim of the cylinder is the same as the acceleration of the hand, together with the angular acceleration of the cylinder, to find the acceleration of the hand.



(a) Apply Newton's 2nd law to the cylinder about an axis through its center of mass:

$$\sum \tau_0 = TR = I_0 \frac{a}{R} \qquad (1)$$

and

$$\sum F_x = Mg - T = 0 \qquad (2)$$

Solve for T to obtain: $T = \boxed{Mg}$

(b) Rewrite equation (1) in terms of α: $TR = I_0\alpha$

Solve for α: $\alpha = \dfrac{TR}{I_0}$

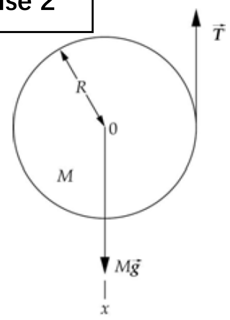
Substitute for T and I_0 to obtain: $\alpha = \dfrac{MgR}{\frac{1}{2}MR^2} = \boxed{\dfrac{2g}{R}}$

(c) Relate the acceleration a of the hand to the angular acceleration of the cylinder: $a = R\alpha$

Substitute for α to obtain: $a = R\left(\dfrac{2g}{R}\right) = \boxed{2g}$

答案 Exercise 2

Picture the Problem The diagram shows the forces acting on the cylinder. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T, a, and α that we can solve simultaneously.



(a) Apply Newton's 2nd law to the cylinder:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum F_x = Mg - T = Ma \qquad (2)$$

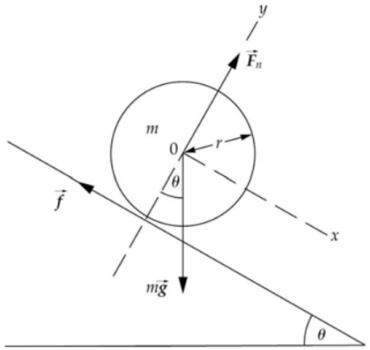
Substitute for α and I_0 in equation (1) to obtain: $TR = \left(\frac{1}{2}MR^2\right)\left(\dfrac{a}{R}\right)$

Solve for T: $T = \frac{1}{2}Ma \qquad (3)$

Substitute for T in equation (2) and solve for a to obtain: $a = \boxed{\frac{2}{3}g}$

(b) Substitute for a in equation (3) to obtain: $T = \frac{1}{2}M\left(\frac{2}{3}g\right) = \boxed{\frac{1}{3}Mg}$

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass equals the net force divided by the mass. The forces acting on the sphere are its weight $m\vec{g}$ downward, the normal force $\vec{F}_n$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the sphere accelerates down the incline, the angular velocity of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_n$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the sphere: $mg\sin\theta - f = ma_{cm}$ (1)

Apply $\sum \tau = I_{cm}\alpha$ to the sphere: $fr = I_{cm}\alpha$

Use the nonslip condition to eliminate α and solve for f: $fr = I_{cm}\dfrac{a_{cm}}{r}$

and

$f = \dfrac{I_{cm}}{r^2}a_{cm}$

Substitute this result for f in equation (1) to obtain: $mg\sin\theta - \dfrac{I_{cm}}{r^2}a_{cm} = ma_{cm}$

From Table 9-1 we have, for a solid sphere: $I_{cm} = \frac{2}{5}mr^2$

Substitute in equation (1) and simplify to obtain: $mg\sin\theta - \frac{2}{5}a_{cm} = ma_{cm}$

Solve for and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{7a_{cm}}{5g}\right) = \sin^{-1}\left[\frac{7(0.2g)}{5g}\right] = \boxed{16.3°}$$

5. $\tau = FR = I\alpha \Rightarrow \alpha = FR/I$

$$\theta = \frac{1}{2}\alpha T^2 = \frac{1}{2}\frac{FR}{I}T^2$$

7. a. $F_{Ha} = Mg/2$　b. $\alpha = 3g/2l$　c. $a_{cm} = 3g/4$　d. $F_{Ha} = Mg/4$

e. $\omega = \sqrt{\dfrac{3g\sin\theta}{l}}$

67 •• A uniform rod of length L_1 and mass M equal to 0.75 kg is attached to a hinge of negligible mass at one end and is free to rotate in the vertical plane (Figure 10-55). The rod is released from rest in the position shown. A particle of mass $m = 0.50$ kg is suspended from a thin string of length L_2 from the hinge. The particle sticks to the rod on contact. What should the ratio L_2/L_1 be so that $\theta_{max} = 60°$ after the collision? SSM

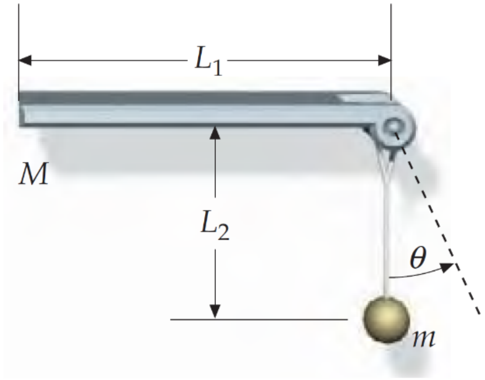


FIGURE 10-55
Problem 67

(APC HW9-2) A round object of mass m, moment of inertia bmr^2 and radius r rolls without slipping along a horizontal surface that then bends upward and backward into an arc of half a circle. The radius of the arc is R.

(a) Derive an expression for the minimum speed of the object's center of mass that will allow it to just pass the top of the arc without losing contact with the track in terms of m, b, r, R and related constants.

The object rolls without slip → conservation of mechanical energy.

Let v_0 be the initial speed and v_f be the final speed passing the top of the arc.

$$\frac{1}{2}mv_0^2+\frac{1}{2}I\omega_0^2=\frac{1}{2}mv_f^2+\frac{1}{2}I\omega_f^2+mg(2R-2r)$$

$$\begin{cases} v_0=\omega_0 r \\ v_f=\omega_f r \end{cases}$$

$$\Rightarrow \frac{1}{2}mv_0^2+\frac{1}{2}bmr^2\left(\frac{v_0}{r}\right)^2=\frac{1}{2}mv_f^2+\frac{1}{2}bmr^2\left(\frac{v_f}{r}\right)^2+mg(2R-2r)$$

$$\frac{1}{2}v_0^2+\frac{1}{2}bv_0^2=\frac{1}{2}v_f^2+\frac{1}{2}bv_f^2+g(2R-2r)$$

$$\frac{1}{2}(1+b)v_0^2=\frac{1}{2}(1+b)v_f^2+g(2R-2r) \qquad (1)$$

At the top of the arc:

$$mg=mv_f^2/(R-r) \Rightarrow v_f^2=g(R-r) \qquad (2)$$

Plug (2) into (1)

$$\frac{1}{2}(1+b)v_0^2=\frac{1}{2}(1+b)g(R-r)+2g(R-r)$$

$$v_0^2=g(R-r)+\frac{4g(R-r)}{1+b} \Rightarrow v_0=\sqrt{g(R-r)+\frac{4g(R-r)}{1+b}}$$

(b) If the object is any of the following one, write the moment of inertia of it respectively.

Hollow cylinder, Solid cylinder, Hollow sphere, Solid sphere.

inertia Equation of Hollow Cylinder	$I = m r^2$
inertia Equation of Solid Cylinder	$I = \frac{1}{2} m r^2$
inertia Equation of Hollow Sphere	$I = \frac{2}{3} m r^2$
inertia Equation of Solid Sphere	$I = \frac{2}{5} m r^2$

(c) Determine the initial speed v_0 needed for the above objects to pass the top of the arc without losing contact with the track from fastest to slowest.

$$v_0^2=g(R-r)+\frac{4g(R-r)}{1+b}$$

The objects in order from fastest to slowest speed needed to make it through the arc are correctly listed as:

solid sphere > solid cylinder > hollow sphere > hollow cylinder.

67 题答案

Use conservation of energy to relate the initial and final potential energy of the rod to its rotational kinetic energy just before it collides with the particle:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}\left(\tfrac{1}{3}ML_1^2\right)\omega^2 + Mg\frac{L_1}{2} - MgL_1 = 0$$

Solving for ω yields:

$$\omega = \sqrt{\frac{3g}{L_1}}$$

Letting ω' represent the angular speed of the rod-and-particle system just after impact, use conservation of angular momentum to relate the angular momenta before and after the collision:

$$\Delta L = L_f - L_i = 0$$

or

$$\left(\tfrac{1}{3}ML_1^2 + mL_2^2\right)\omega' - \left(\tfrac{1}{3}ML_1^2\right)\omega = 0$$

Solve for ω' to obtain:

$$\omega' = \frac{\tfrac{1}{3}ML_1^2}{\tfrac{1}{3}ML_1^2 + mL_2^2}\,\omega$$

Use conservation of energy to relate the rotational kinetic energy of the rod-plus-particle just after their collision to their potential energy when they have swung through an angle θ_{max}:

$$K_f - K_i + U_f - U_i = 0$$

Because $K_f = 0$:

$$-\tfrac{1}{2}I\omega'^2 + Mg\left(\tfrac{1}{2}L_1\right)\left(1-\cos\theta_{max}\right) + mgL_2\left(1-\cos\theta_{max}\right) = 0 \qquad (1)$$

Express the moment of inertia of the system with respect to the pivot:

$$I = \tfrac{1}{3}ML_1^2 + mL_2^2$$

Substitute for θ_{max}, I and ω' in equation (1):

$$\frac{3\dfrac{g}{L_1}\left(\tfrac{1}{3}ML_1^2\right)^2}{\tfrac{1}{3}ML_1^2 + mL_2^2} = Mg\left(\tfrac{1}{2}L_1\right) + mgL_2$$

Simplify to obtain:

$$L_1^3 = 2\frac{m}{M}L_1^2L_2 + 3\frac{m}{M}L_2^2L_1 + 6\frac{m}{M}L_2^3$$

Dividing both sides of the equation by L_1^3 yields:

$$1 = 2\left(\frac{m}{M}\right)\left(\frac{L_2}{L_1}\right) + 3\left(\frac{m}{M}\right)\left(\frac{L_2}{L_1}\right)^2 + 6\left(\frac{m}{M}\right)^2\left(\frac{L_2}{L_1}\right)^3$$

Let $\alpha = m/M$ and $\beta = L_2/L_1$ to obtain:

$$6\alpha^2\beta^3 + 3\alpha\beta^2 + 2\alpha\beta - 1 = 0$$

Substitute for α and simplify to obtain the cubic equation in β:

$$8\beta^3 + 6\beta^2 + 4\beta - 3 = 0$$

Use the solver function of your calculator to find the only real value of β:

$$\beta = \boxed{0.39}$$

(APC HW9-2) A round object of mass m, moment of inertia bmr² and radius r rolls without slipping along a horizontal surface that then bends upward and backward into an arc of half a circle. The radius of the arc is R.

(a) Derive an expression for the minimum speed of the object's center of mass that will allow it to just pass the top of the arc without losing contact with the track in terms of m, b, r, R and related constants.

The object rolls without slip → conservation of mechanical energy.

Let v_0 be the initial speed and v_f be the final speed passing the top of the arc.

$$\frac{1}{2}mv_0^2+\frac{1}{2}I\omega_0^2=\frac{1}{2}mv_f^2+\frac{1}{2}I\omega_f^2+mg(2R-2r)$$

$$\begin{cases} v_0=\omega_0 r \\ v_f=\omega_f r \end{cases}$$

$$\Rightarrow \frac{1}{2}mv_0^2+\frac{1}{2}bmr^2\left(\frac{v_0}{r}\right)^2=\frac{1}{2}mv_f^2+\frac{1}{2}bmr^2\left(\frac{v_f}{r}\right)^2+mg(2R-2r)$$

$$\frac{1}{2}v_0^2+\frac{1}{2}bv_0^2=\frac{1}{2}v_f^2+\frac{1}{2}bv_f^2+g(2R-2r)$$

$$\frac{1}{2}(1+b)v_0^2=\frac{1}{2}(1+b)v_f^2+g(2R-2r) \qquad (1)$$

At the top of the arc:

$$mg=mv_f^2/(R-r) \ \Rightarrow\ v_f^2=g(R-r) \qquad (2)$$

Plug (2) into (1)

$$\frac{1}{2}(1+b)v_0^2=\frac{1}{2}(1+b)g(R-r)+2g(R-r)$$

$$v_0^2=g(R-r)+\frac{4g(R-r)}{1+b} \ \Rightarrow\ v_0=\sqrt{g(R-r)+\frac{4g(R-r)}{1+b}}$$

(b) If the object is any of the following one, write the moment of inertia of it respectively.

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(c) Determine the initial speed v_0 needed for the above objects to pass the top of the arc without losing contact with the track from fastest to slowest.

$$v_0^2=g(R-r)+\frac{4g(R-r)}{1+b}$$

The objects in order from fastest to slowest speed needed to make it through the arc are correctly listed as:

solid sphere > solid cylinder > hollow sphere > hollow cylinder.

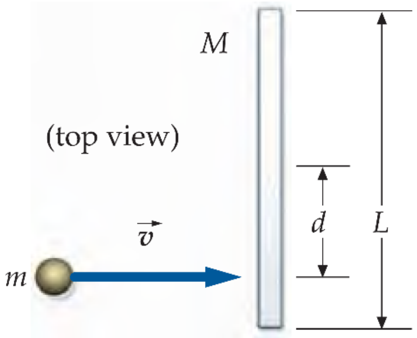
问题分析:

$$v_f^2=g(R-r) \qquad (2)$$

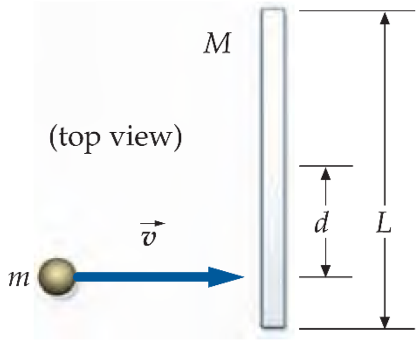
v_f相同

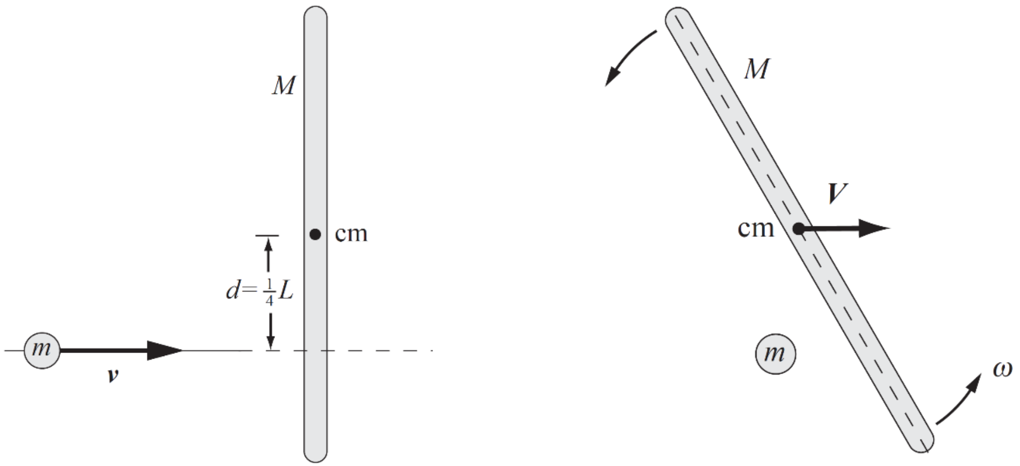
$$\frac{1}{2}(1+b)v_0^2=\frac{1}{2}(1+b)v_f^2+g(2R-2r) \qquad (1)$$

EX62: The right figure shows a thin, uniform bar whose length is L and mass is Mand a compact hard sphere whose mass is m. The system is supported by a frictionless horizontal surface. The sphere moves to the right with velocity $\vec{v}$, and strikes the bar at a distance $\frac{1}{4}L$ from the center of the bar. The collision is elastic, and following the collision the sphere is at rest. Find the value of the ratio m/M.



EX62: The right figure shows a thin, uniform bar whose length is L and mass is Mand a compact hard sphere whose mass is m. The system is supported by a frictionless horizontal surface. The sphere moves to the right with velocity $\vec{v}$, and strikes the bar at a distance $\frac{1}{4}L$ from the center of the bar. The collision is elastic, and following the collision the sphere is at rest. Find the value of the ratio m/M.





设碰撞后 M 的速度为 V,

碰撞前后动量守恒: $mv = MV \Rightarrow \quad V = \frac{m}{M}v$ (1)

碰撞前后角动量守恒: $mv\left(\frac{1}{4}L\right) = I_{cm}\omega = (\frac{1}{12}ML^2)\omega$ (2)

弹性碰撞能量守恒: $\frac{1}{2}mv^2 = \frac{1}{2}MV^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}MV^2 + \frac{1}{2}(\frac{1}{12}ML^2)\omega^2$ (3)

(2) $\Rightarrow \omega = \frac{3mv}{ML}$ (4)

(3) $\Rightarrow mv^2 = MV^2 + (\frac{1}{12}ML^2)\omega^2$

Plug in (1) and (4) $\Rightarrow mv^2 = M(\frac{m}{M}v)^2 + (\frac{1}{12}ML^2)(\frac{3mv}{ML})^2$

$\Rightarrow v^2 = \frac{m}{M}v^2 + \frac{3}{4}\frac{mv^2}{M} = \frac{7}{4}\frac{m}{M}v^2$

$\Rightarrow \frac{m}{M} = \frac{4}{7}$

63 •• Figure 10-53 shows a uniform rod whose length is L and mass is M pivoted at the top. The rod, which is initially at rest, is struck by a particle whose mass is m at a point $x = 0.8L$ below the pivot. Assume that the particle sticks to the rod. What must be the speed v of the particle so that following the collision the maximum angle between the rod and the vertical is 90°?

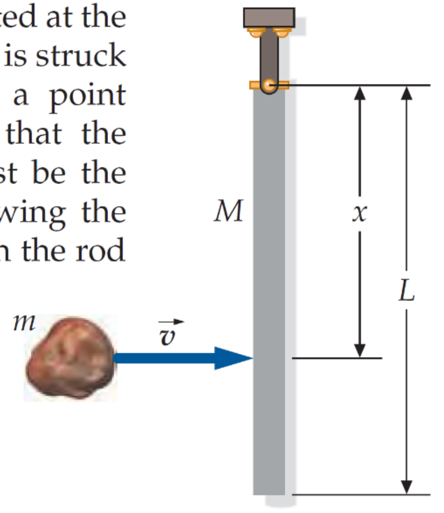


FIGURE 10-53
Problem 63

63：　设碰撞后角速度为ω

角动量守恒：　$mv(0.8L) = [I + m(0.8L)^2]\omega = \left(\frac{1}{3}ML^2 + 0.64mL^2\right)\omega$　(1)

能量守恒：　$\frac{1}{2}I\omega^2 + \frac{1}{2}m[\omega(0.8L)]^2 = Mg\left(\frac{1}{2}L\right) + mg(\frac{1}{2}0.8L)$　(2)

$(1) \Rightarrow\ \omega = \frac{0.8mv}{\frac{1}{3}ML+0.64mL}$

$(2) \Rightarrow\ (\frac{1}{3}ML^2)\omega^2 + 0.64m\omega^2L^2 = MgL + 0.8mgL$

$\Rightarrow\ \left[\left(\frac{1}{3}ML\right) + 0.64mL\right]\omega^2 = Mg + 0.8mg$

$\Rightarrow\ \omega^2 = \frac{Mg+0.8mg}{\left(\frac{1}{3}ML\right)+0.64mL} = \left(\frac{0.8mv}{\frac{1}{3}ML+0.64mL}\right)^2$

$$Mg + 0.8mg = \frac{0.64(mv)^2}{\frac{1}{3}ML + 0.64mL}$$

$$v^2 = \frac{(Mg + 0.8mg)\left(\frac{1}{3}ML + 0.64mL\right)}{0.64m^2}$$

$$v = \sqrt{\frac{(Mg + 0.8mg)\left(\frac{1}{3}ML + 0.64mL\right)}{0.64m^2}}$$